MATH 4242 Quiz 9

Name:_____
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Let $A = [2, 1]$. What is the spectral decomposition of $K = A^t A$. Use this to find the Singular value decomposition of A.

Proof. $K = A^t A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$. The characteristic polynomial is $x^2 - 5x$, so there are two eigenvalues $\lambda_1 = 5$ and $\lambda_2 = 0$, with eigenvectors $v_1 = (2, 1)$ and $v_2 = (-1, 2)$. To find the spectral decomposition of K, we need to turn v_1, v_2 into unit vectors.

So $K = Q \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix} Q^t$ where $Q = \begin{pmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ -1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix}$.

The SVD for A is $A = P \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} Q^t$ where $P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

Alternatively, $A = P\Sigma Q^t$ where $P = (1)$, $\Sigma = (\sqrt{5})$ and $Q^t = (2/\sqrt{5}, 1/\sqrt{5})$ □