MATH 4242 Quiz 8

Name:________________
Student Id:________________

Find a Jordan basis for $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ and write down the Jordan decomposition.

Proof. The characteristic polynomial is $(x-1)^3$. So it has only 1 eigenvalue.

We will find a Jordan chain w_1, w_2, w_3 for A. The first vector w_1 should be an eigenvector.

$\ker(A-I) = \ker \begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} = \{(x,0,0) | x \in \mathbb{R}\}$. So $(1,0,0)$ is an eigenvector, and let $w_1 = (1,0,0)$.

Next we find w_2:

$$\begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} (x,y,z)^t = (1,0,0)^t \implies y = \frac{1}{2},\ z = 0.$$ Thus we can let $w_2 = (0, 1/2, 0)$.

Next we find w_3:

$$\begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} (x,y,z)^t = (0,1/2,0)^t \implies y = -\frac{3}{8},\ z = \frac{1}{4}.$$ Thus we can let $w_2 = (0, -3/8, 1/4)$.

So

$$A = K \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} K^{-1} \text{ where } K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -3/8 \\ 0 & 0 & 1/4 \end{pmatrix}$$

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