MATH 4242 Quiz 6

Name:_____
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Let $A = \begin{pmatrix} 5 & -3 \\ 6 & -4 \end{pmatrix}$.

Is A diagonalizable? If so, find K and D such that $A = K^{-1}DK$ where D is a diagonal matrix.

Proof. The characteristic polynomial is $x^2 - x - 2 = (x+1)(x-2)$. So there are two distinct eigenvalues, thus the matrix is diagonalizable.

Next we find the eigenvectors.

For $\lambda = 2$:

$\ker(A - 2I) = \ker \begin{pmatrix} 3 & -3 \\ 6 & -6 \end{pmatrix} = \ker \begin{pmatrix} 3 & -3 \\ 0 & 0 \end{pmatrix} = \{(x, x) : x \in \mathbb{R}\} = \text{span}(1, 1)$.

For $\lambda = -1$:

$\ker(A - (-1)I) = \ker \begin{pmatrix} 6 & -3 \\ 6 & -3 \end{pmatrix} = \ker \begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix} = \{(x, 2x) | x \in \mathbb{R}\} = \text{span}(1, 2)$.

So $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$

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