

MATH 4242 Quiz 3

Name: _____
Student Id: _____

Consider the map $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ defined by $T(x, y, z) = x + y + z$.

- (1) Is T linear? If so, what is $\mathcal{M}(T)$? [6pts]

Proof. It's easy to check that T is linear. To find the matrix, consider how T acts on basis of \mathbb{R}^3 . Let e_1, e_2, e_3 be the standard basis.

$T(e_1) = 1, T(e_2) = 1, T(e_3) = 1$. And 1 is the basis of \mathbb{R} . So

$$\mathcal{M}(T) = [1, 1, 1] \quad \square$$

- (2) Describe the kernel of T . [4pts]

$$\{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\} = \{x, y, -x - y \mid x, y \in \mathbb{R}\}$$

- (3) What's the dimension of $\text{Img}(T)$? [2pts, extra credit]

The map T is surjective, because every number a in \mathbb{R} , we have that $T(a, 0, 0) = a$. Therefore the image of T is \mathbb{R} , whose dimension is 1. (Can also prove using the fact that $\dim \ker(T) = 2$)