MATH 4242 Quiz 2

Name:________________
Student Id:________________

(1) Prove or provide a counterexample. Does $\{(1,1),(2,5)\}$ form a basis of $\mathbb{R}^2$? (3 pts)

Proof. Yes. First we show that $(1,1)$ and $(2,5)$ is linearly independent, i.e. the only way to write $(0,0)$ as a linearly combination of them is using 0 coefficient. This is equivalent to solving the system of equations.

$$x+y=0$$
$$2x+5y=0$$

Using Gaussian elimination, we found only one solution which is $x=y=0$. Therefore they are linearly independent.

We know that $\dim\mathbb{R}^2=2$, and linearly independent list of vectors of the correct size for a basis, so $(1,1),(2,5)$ form a basis of $\mathbb{R}^2$. □

(2) Are the vectors $(1,3),(0,-1),(1,1)\in\mathbb{R}^2$ linearly independent? Explain your answer. (3 pts)

Proof. No. $\mathbb{R}^2$ has dimension two, so every basis of $\mathbb{R}^2$ has size two. But we know that if the three vectors were linearly independent, then they extend to a basis of $\mathbb{R}^2$ of at least three vectors[1], which is a contradiction. □

(3) What's the dimension of $V=\{(x,y,z)|x+y+z=0\}$, as a subspace of $\mathbb{R}^3$? (4 pts)

Proof. V an be re-written as

$$V=\{(x,y,-x-y)|x,y\in\mathbb{R}\}$$

Every vector in V of the form $(a,b,-a-b)$ can be uniquely written as $a(1,0,-1)+b(0,1,-1)$. Therefore $V=\operatorname{span}\{(1,0,-1),(0,1,-1)\}$. These two vectors are linearly independent, so they form a basis of V. Thus $\dim V=2$. □

[1]Using the lemma that linearly independent list of vectors extends to a basis.